On the Exit Boundary Condition for One-Dimensional Calculations of Pulsed Detonation Engine Performance

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Abstract

In one-dimensional calculations of pulsed detonation engine (PDE) performance, the exit boundary condition is frequently taken to be a constant static pressure. In reality, for an isolated detonation tube, after the detonation wave arrives at the exit plane, there will be a region of high pressure, which will gradually return to ambient pressure as an almost spherical shock wave expands away from the exit, and weakens. Initially, the flow is supersonic, unaffected by external pressure, but later becomes subsonic. Previous authors have accounted for this situation either by assuming the subsonic pressure decay to be a relaxation phenomenon, or by running a two-dimensional calculation first, including a domain external to the detonation tube, and using the resulting exit pressure temporal distribution as the boundary condition for one-dimensional calculations. These calculations show that the increased pressure does affect the PDE performance.

In the present work, a simple model of the exit process is used to estimate the pressure decay time. The planar shock wave emerging from the tube is assumed to transform into a spherical shock wave. The initial strength of the spherical shock wave is determined from comparison with experimental results. Its subsequent propagation, and resulting pressure at the tube exit, is given by a numerical blast wave calculation. The model agrees reasonably well with other, limited, results.

Finally, the model was used as the exit boundary condition for a one-dimensional calculation of PDE performance to obtain the thrust wall pressure for a hydrogen-air detonation in tubes of length to diameter ratio (L/D) of 4, and 10, as well as for the original, constant pressure boundary condition. The modified boundary condition had no performance impact for values of L/D > 10, and moderate impact for L/D = 4.

Introduction

One-dimensional CFD calculations are very useful in assessing the effects of parameter variations quickly, since they can usually be performed quite rapidly. However, the flow emerging from a detonation tube becomes three-dimensional immediately; some way of approximating this three-dimensional situation is needed to establish the boundary condition at

the tube end. At some point in the cycle of a pulsed detonation engine, a strong compression wave will arrive at the exit of the tube, and propagate into the region beyond the detonation tube. This wave will either be the detonation wave itself, if the tube is completely filled with combustible mixture, or the transmitted shock from the interaction of the detonation wave with the combustible gas-air interface. In calculating the cycle, it is necessary to know how this wave reflects at the tube exit. For weak waves, it is well known that a shock wave reflects at the exit of a tube as an expansion wave, with the exit pressure approximately constant (Rudinger 1955a). This result is so ingrained that it is tempting to use it even for strong shocks such as those found in the pulsed detonation engine. However, this is not true for strong shocks. Rudinger (1955a) concluded that in this case, if the outflow is supersonic, and since pressure waves cannot travel upstream in supersonic flow, the pressure cannot return to ambient conditions. He states that "final expansion to the exterior pressure must then take place outside the duct, and is of no concern here." Despite this, Rudinger (1955b) subsequently, using acoustic theory, calculated the pressure decay as a function of time at the end of a shock tube following the exit of a shock wave from the tube. He found that the pressure decays in the time in which a sound wave can travel about three tube diameters. The calculation was in quite good agreement with values inferred from upstream pressure measurements in a shock tube. The calculation is valid only for weak shock waves, but even in this case the pressure does not return to atmospheric immediately after the emergence of a shock wave from a tube. The appropriate dimension for the pressure decay is the tube diameter, not its length.

For the calculation of a PDE cycle, it is necessary to estimate the pressure decay rate as it can affect the calculated performance. Considered here will be the case of an isolated detonation tube, which, though not necessarily representative of an actual engine, is typical of many experiments. Kailasanath (2001) treated this problem by assuming that, after the flow becomes subsonic, the exit pressure decayed as a relaxation process, and found that the higher pressure at the tube exit increased the PDE performance. However, it is not clear what relaxation time should be used. Ebrahimi et al. (2000) performed two-dimensional calculations, including a region external to the tube, to establish a pressure-time relationship at the tube exit to use in one-dimensional calculations. These two-dimensional calculations must be repeated for calculating a different geometry, so this destroys the ease of use of one-dimensional calculations. What is needed is a simple way of applying a two-dimensional (or more) result to a one-dimensional calculation. This is the objective of the present work, and is achieved by using a model of the external flow.

Symbols

a	speed of sound
D	diameter of detonation tube
E	energy of shock wave
L	length of detonation tube
ℓ	length filled with reactive mixture
P	Riemann invariant = $u + 2\,a/(\gamma - 1)$
p	pressure
Δp	pressure jump at shock wave
Q	detonation enthalpy/unit mass
R	radial distance from shock wave center

t	real time
u	gas velocity
V_{cj}	detonation wave velocity
x	axial distance from end of tube
γ	ratio of specific heats
ε	length related to shock wave energy
λ	non-dimensional radius $= R/\varepsilon$
ρ	density
τ	nondimensional time

Superscripts:

chem	chemical
PDE	related to pulsed detonation device
ref	reference value
s	value at shock wave
0	initial position of spherical shock
2	ambient air
1,3,4,5	regions defined in fig. 1

Model Description

The planar wave leaving the exit of the tube will be the transmitted wave from the interaction of the detonation wave on the mixture/air interface (assuming a fuel-air reaction). The solution to the problem of a shock (or detonation) wave incident on an interface is given by Rudinger (1995a). The scheme is illustrated in fig. 1, which is a space–time diagram showing wave trajectories. Region 1 is assumed to be filled with a detonable mixture of fuel and air, with the fuel being hydrogen in the example. Region 2 is assumed to be air, although in a situation of repeated detonations, it might be vitiated air. The interface between the two gases cannot support a pressure difference, and so velocity and pressure are the same in both regions. For a single detonation, the velocity will be zero, as in fig. 1, but may not be for repeated detonations.
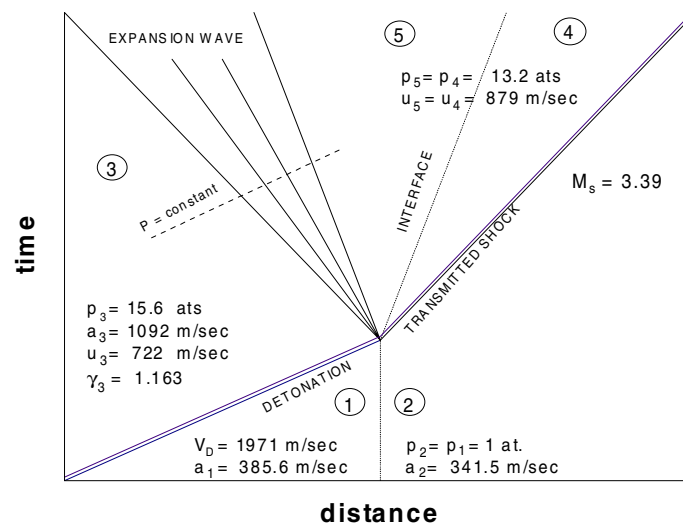

Figure 1. Distance-time plot of the interaction of a detonation wave with an interface.

The detonation wave impinges on the interface from the left. The gas on the right hand side of the interface contains no fuel, and so will not support a detonation, and the transmitted wave will be a shock wave. The conditions behind a hydrogen-air detonation are given in Borman and Ragland (1998), and are listed in fig. 1. The process of solving for the transmitted shock strength is iterative. A shock Mach number is assumed, from which the pressure and velocity behind the shock (region 4) can be calculated. These values must also hold in region 5, to the left of the interface. Given the pressure ratio across the expansion wave, the temperature ratio can be calculated since an expansion wave is isentropic. Hence the speed of sound in region 5 can be determined. Across an expansion, the Riemann invariant, $P = u + 2 a / (\gamma - 1)$ is constant, so another evaluation of the velocity in region 5 can be made by equating the known P_3 to P_5. The procedure is repeated until both values of u_5 agree. Using a spreadsheet, the iteration can be performed rapidly.

With this procedure, the transmitted shock in air is found to have a Mach number of 3.385, with a pressure ratio of 13.2. This is the initial pressure at the exit after the shock emerges. Although this is a one-dimensional result, it will hold until expansion waves from the edges have reduced the pressure. The development of the shock wave is envisioned in fig. 2. On leaving the detonation tube, the wave is still planar, except at the edges (fig. 2a). As the edge waves grow, the shock wave becomes more spherical, although initially there will still be a planar portion near the axis (fig. 2b). At later time, the wave will become essentially spherical (fig. 2c).

That this actually occurs can be seen from the photographs of a similar situation, namely, the precursor blast wave from a gun (Heimerl and Klingenberg 1983), showing an almost spherical shock wave propagating ahead of the bullet. Schlieren photographs of a shock wave emerging from an open shock tube (Elder and de Haas 1952) add further confirmation.
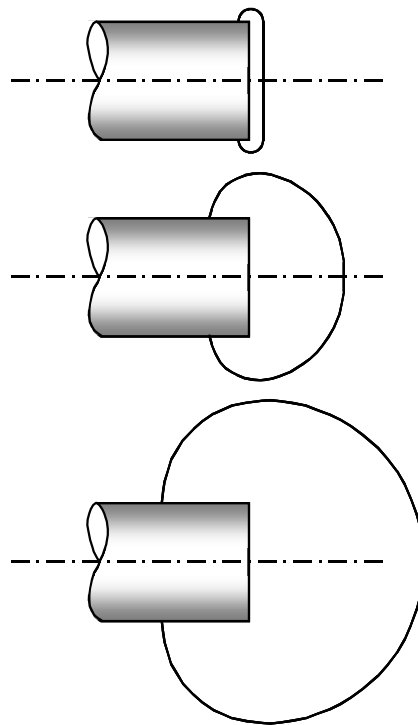

Figure 2. (a) the shock wave emerging from the detonation tube is mostly planar,
(b) at a distance of about 0.7 D, only a small planar region remains,
(c) for distances above 1.3 D, the wave is almost spherical.

For calculating pressures it is necessary to ascertain the strength of the spherical shock wave. For this, recourse is made to the experiments of Ungut et al. (1984). These workers measured the centerline trajectory of the transmitted wave from detonations in various mixtures in a tube of diameter D, expanding into a larger region, which also contained the combustible gas mixture. The centerline shock velocity stayed almost constant, consistent with unattenuated planar shock propagation, up to a distance of about 0.7 D from the tube exit. In cases in which detonation was not reinitiated in the larger region, the wave then decelerated rapidly to a distance of 1.3 D, after which it decelerated more slowly. This is shown in fig. 3, showing two of the trajectories measured by Ungut et al., for two different mixtures, in tubes of different diameters. At the point at which the deceleration decreases, the velocity of the shock wave is seen to be about half the Chapman-Jouguet detonation velocity V_{cj}. This will be taken to be a general rule, namely that the spherical shock is characterized by having a velocity of half V_{cj} at a distance of 1.3 diameters from the tube exit, at which point the shock radius is also 1.3 D. The shock radius of 1.3D will be called the initial radius, R_0, of the spherical wave. Thus the spherical wave pressure jump at R_0, Δp_0, will be that corresponding to a shock wave traveling at half the detonation velocity. This defines the spherical blast wave. For hydrogen-air, with a detonation velocity of 1971 m/sec, $\Delta p_0 / p_2 = 8.58$. From this point on, the wave will be considered to propagate as a spherical blast wave. The propagation of a spherical blast wave into an ambient pressure of one atmosphere has been evaluated numerically by Brode (1955). Brode calculated the entire pressure distribution behind the blast wave for 32 different cases, of increasing shock radius. Three examples are given in fig. 4, in which pressure is plotted against dimensionless radius $\lambda = R/\varepsilon$, where $\varepsilon = (E/p_2)^{1/3}$ is a length determined by the energy E which produced the shock wave, and the ambient pressure p_2. A strong shock is shown at $\lambda = 0.28$, for which the pressure behind the wave decreases monotonically with decreasing radius, reaching a value at the
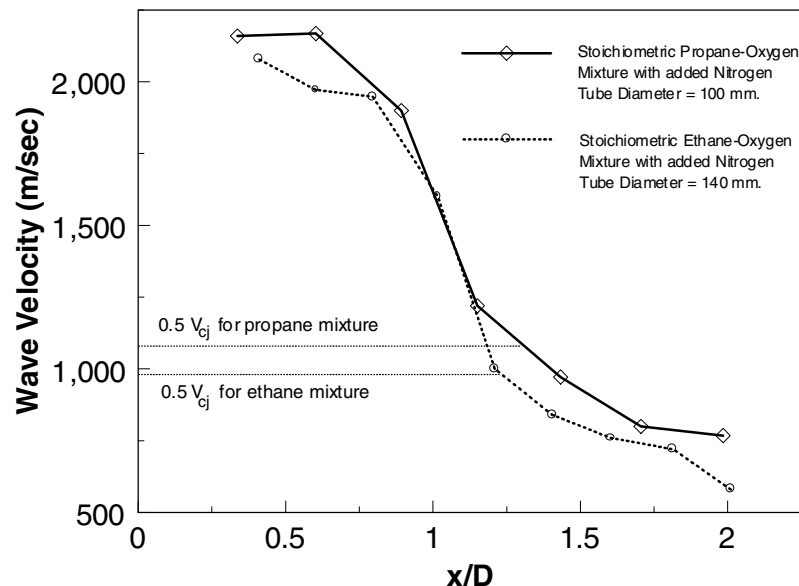

Figure 3. Shock wave speed versus non-dimensionalized distance from the end of the detonation tube (x/D), from the experiments of Ungut et al. (1984).
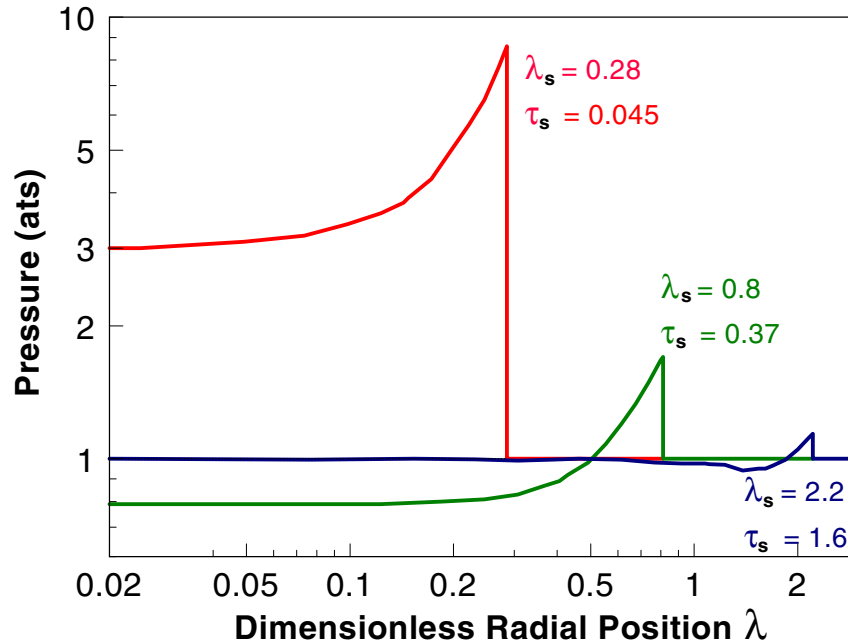
Figure 4. Pressure distribution behind blast waves for shocks
at three dimensionless radii, from Brode (1955).

center of the wave system p(R= 0) of 0.375 of the pressure behind the shock front. For the medium strength shock at $\lambda = 0.8$, the pressure at wave center is below atmospheric. The weak shock at $\lambda = 2.2$ is followed by a region of sub-atmospheric pressure, but the pressure returns to atmospheric at the wave center. The value of the pressure at the wave center will be considered to be the pressure boundary condition at the detonation tube exit, which is the objective of this work. Thus the important values from Brode's work are the shock wave pressure ratio, the pressure at the wave center, and the time and radius at which these occur. Taken from Brode's graphs, these quantities are given in table 1. A given shock pressure jump $\Delta p/p_2$ will occur at a dimensionless shock radius $\lambda_s = R_s/\varepsilon$, according to the relation:

$$\Delta p/p_2 = 0.137/\lambda_s^3 + 0.119/\lambda_s^2 + 0.269/\lambda_s - 0.019 \qquad (1)$$

For the detonation case, since $\Delta p_0/p_2$ is known when the shock is at a radius $R_s = R_0 = 1.3$ D, the value of λ_s follows from the above equation, and hence the value of ε can be determined. For hydrogen-air detonations, with $\Delta p_0/p_2 = 8.58$, $\lambda_s = 0.282$, and $\varepsilon = 4.61$ D.

The pressure at the wave center, p(R = 0) is given by Brode in terms of a dimensionless time $\tau = t \, a_2 / \varepsilon$, in which t = real time, and a_2 is the speed of sound in the ambient air, and is plotted in fig. 5. The Brode portion of this graph is a universal plot, and constitutes the desired boundary condition for the detonation tube exit. For any particular case, ε is found as described above, and then τ can be converted to real time.

Table 1. Properties of a blast wave, from Brode (1955).

| Shock Pressure Ratio	Dimensionless Time	Dimensionless Radius	$p(R=0)/p_2$
11	0.0350	0.258	4.0
8.6	0.0448	0.286	3.0
7.0	0.0565	0.320	2.4
5.2	0.0761	0.360	1.8
4.0	0.0995	0.406	1.4
3.2	0.1307	0.467	1.15
3.15	0.1386	0.479	1.1
2.8	0.1620	0.535	0.99
2.41	0.2011	0.575	0.90
2.13	0.2479	0.645	0.83
1.92	0.2948	0.707	0.80
1.70	0.3709	0.811	0.79
1.52	0.4979	0.971	0.80
1.41	0.6229	1.112	0.86
1.39	0.6542	1.125	0.86
1.28	0.8729	1.383	0.92
1.20	1.1854	1.721	0.97
1.14	1.6229	2.212	1.00
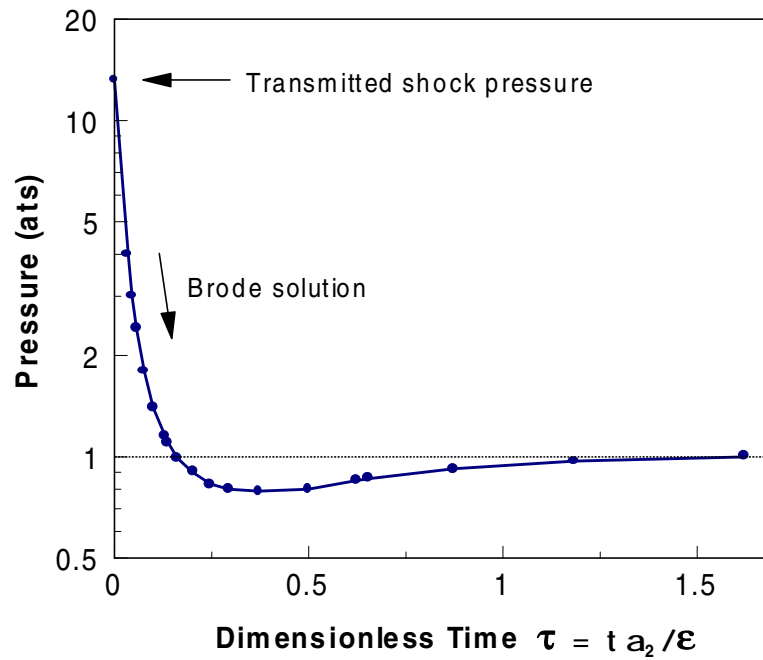
Figure 5. Pressure at the blast wave center versus dimensionless time.

Comparison with Other Results

Moen et al. (1982) have reported measurements of the overpressure at three distances from the exit of the Norwegian large explosion experiment, and also report a calculation of overpressure versus distance by Hjertager for a methane-air detonation, although detonation in methane-air was not apparently achieved. For methane-air, the detonation velocity is 1801 m/sec, and hence $\Delta p_0/p_2 = 6.96$. For this experiment, the tube length is 10m, and the diameter is 2.5 m, giving an L/D of 4. With these values, the overpressure versus distance can be evaluated using the above model. The result of this calculation, together with Hjertager's calculation are plotted in fig. 6a. The distance to a given overpressure for the blast wave calculation is about 80% of the values found by Hjertager. This is quite good agreement. There do not appear to be any experimental data for the detonation case. However, the Hjertager calculation does give a good fit to the available data for explosions without detonation, and so is presumably reliable also for detonations.

Sochet et al. (1999) generated planar detonations in a cylindrical half tube, closed at one end, and open at the other. The tube was mounted on a plane. They used pressure transducers to measure the time taken by the blast wave emerging from the open end into the atmosphere to reach seven locations in the plane along the cylinder axis. Their results for a stoichiometric hydrogen-oxygen mixture in a 16 mm diameter half tube are given in fig. 6b, together with calculations using the present blast wave model. The agreement is excellent. Not quite so good is the agreement for detonation of a propane-oxygen mixture in a 36 mm tube, but it is still not bad. That the blast wave calculation is in fair agreement with the large scale, i.e., 50 m blast wave radius calculation for the Norwegian experiment, and good agreement with the small scale, i.e., less than 0.4 m blast wave radius, experiment of Sochet et al., is very encouraging.
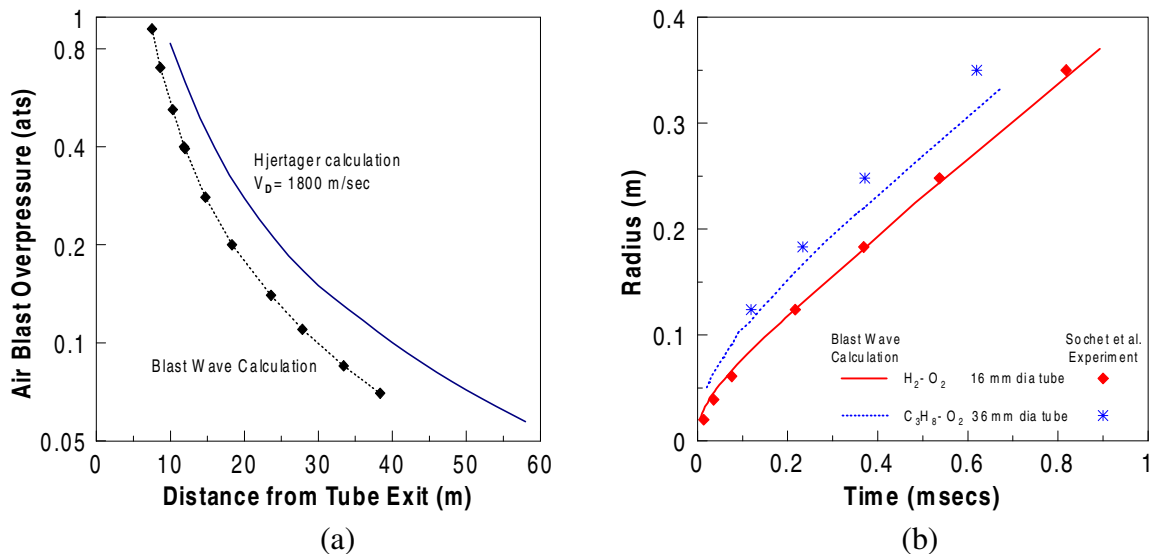

(a)

(b)

Figure 6. (a) Calculated blast overpressure versus distance from the blast wave calculation, and from Hjertager's calculation (Moen et al. 1982), for the Norwegian large explosion experiment. (b) Plot of blast wave radius versus time after leaving the detonation tube using the blast wave calculation, together with the experimental points of Sochet et al. (1999).

Application to PDE Calculation

Ebrahimi et al. (2000) performed CFD calculations of the pressure distribution at the end of a 20 mm high rectangular detonation tube, assuming a cylindrical blast wave, by including a region external to the detonation tube into the calculation. Since a cylindrical blast wave will decay at a slower rate than a spherical one, there is not a direct comparison here, but there should be qualitative agreement. A comparison of the prediction of the present model with that of Ebrahimi et al. is given in fig. 7. As will be seen, there is indeed qualitative agreement, with the blast wave calculation decaying more rapidly as expected. Note, however, that if the velocity of the gas exiting from the detonation tube is sonic or greater, the internal pressure at the end of the tube should differ from the external pressure. The acoustic calculation of Rudinger (1955b) is also given. Rudinger's calculation is in quite good agreement with his experimental evaluations of internal pressure at the tube exit, and is close to the result of Ebrahimi et al. (2000).
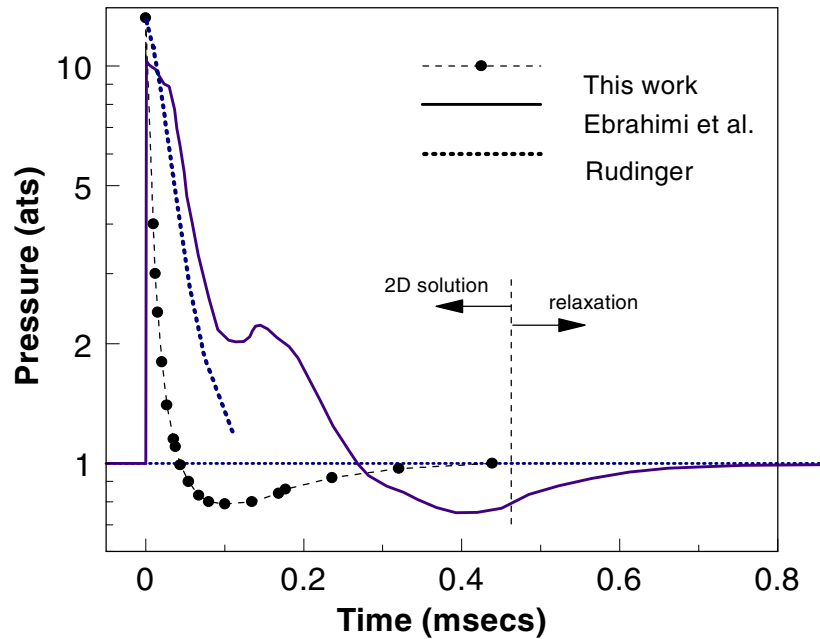

Figure 7. Comparison of the external pressure evolution with time for the blast wave calculation for a 20 mm tube, and the CFD calculation of Ebrahimi et al. The acoustic calculation of Rudinger is also indicated.

A one-dimensional, time accurate, CFD code for analysis of PDE cycles has been developed at the NASA Glenn Research Center by Paxson (2001). Using a high-resolution scheme, the code numerically solves the governing equations for a reacting, two species (single progress variable), calorically perfect gas with specified boundary conditions. In the code, a non-dimensional time is used, defined as:

$$\tau_{PDE} = a_{ref} \, t \, / \, L \tag{2}$$

where a_{ref} is some appropriate speed of sound, and L is the length of the device. For hydrogen-air detonations, $\varepsilon = 4.61 \, D$, from which

$$\tau = \tau_{PDE} \, (\, L/D \,) \, (a_2 / a_{ref}) / 4.61 \tag{3}$$

in which τ must be measured from the arrival of the detonation wave, or transmitted wave, at the tube exit. At any value of τ, the exit pressure can be found from fig. 5, and is assumed to be imposed at the exhaust boundary of the tube. Using this pressure, and the known state of the gas in the last interior computational cell, a so-called Half-Riemann problem can be solved giving rise to two possible states separated by an associated wave and a contact discontinuity. This is illustrated in fig. 8. If the solution to this problem results in a strictly left running wave (e.g., a subsonic shock or fan) as shown in the figure, then the pressure in the region labeled 2 is identical to the imposed external pressure. If, on the other hand, the solution yields a right running wave (e.g., a shock followed by supersonic flow behind it, or a fan that crosses the sonic line) then the flow in Region 2 will not be at the same pressure as the imposed pressure. Then the conditions in Region 2 are entirely determined by the interior cell state. The pressure trace labeled 'Internal static pressure' in fig. 9 represents the computed conditions in Region 2.

Calculations for tube L/D ratios of 4, and 10, as well as a calculation with the constant pressure boundary condition, have been made, and the results are shown in fig. 9 and fig. 10. Figure 9 shows the calculated internal static pressure at the end of the tube, as well as the external pressure calculated with the blast wave model, and the Mach number of the flow at the tube exit. As can be seen, the Mach number is above unity up to a τ_{PDE} of 0.83, and hence the flow is choked and can not be affected by external pressure. With a tube of L/D = 10 (fig. 9a), the flow static pressure and external pressure are both equal, and equal to one atmosphere at τ_{PDE} = 0.83. Thus by the time that the external pressure could have any effect, it has already returned to one atmosphere. Obviously a boundary condition of constant pressure equal to one atmosphere has the same result. Calculation confirmed this, and so the result is not plotted. Any value of L/D above 10 will give the same result as the constant, one atmosphere, boundary condition. For a tube of L/D of 4, as shown in fig. 9b, the external pressure, and the tube static pressure, which is equal to the external pressure, are both sub-atmospheric at τ_{PDE} = 0.83, though not by much. However, it is enough to affect the cycle. This can be seen in fig. 10, which is the evolution with time of the pressure on the front, i.e., thrust, wall. The top of the leading edge of the positive pressure pulse at the front wall for L/D = 4 is delayed relative to that for L/D = 10, and the integral of pressure with time is reduced, indicating somewhat lower thrust. This effect will be increased for lower values of L/D, but since such values do not seem practical, no such results are given. Note, however, that the limiting value of L/D, above which no difference from the constant pressure boundary condition is seen, is dependant on the assumption of the model that R_0 = 1.3 D. As was seen in fitting the blast wave model to Hjertager's calculation (Moen 1982), the blast wave model gave lower values of radius for a given overpressure than did Hjertager. Better agreement could be reached by using a larger value for R_0. This would lead to a larger value of the limiting L/D for detonation tube calculations. On the other hand, use of a larger value of R_0 appears inconsistent with the data of Ungut et al. (1984), and Sochet et al. (1999). In any case, a more exact value of R_0 is not likely to influence the present conclusions greatly.
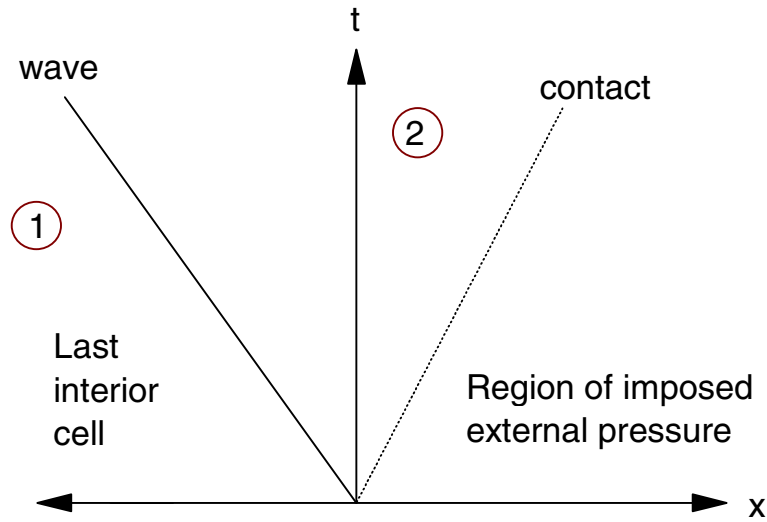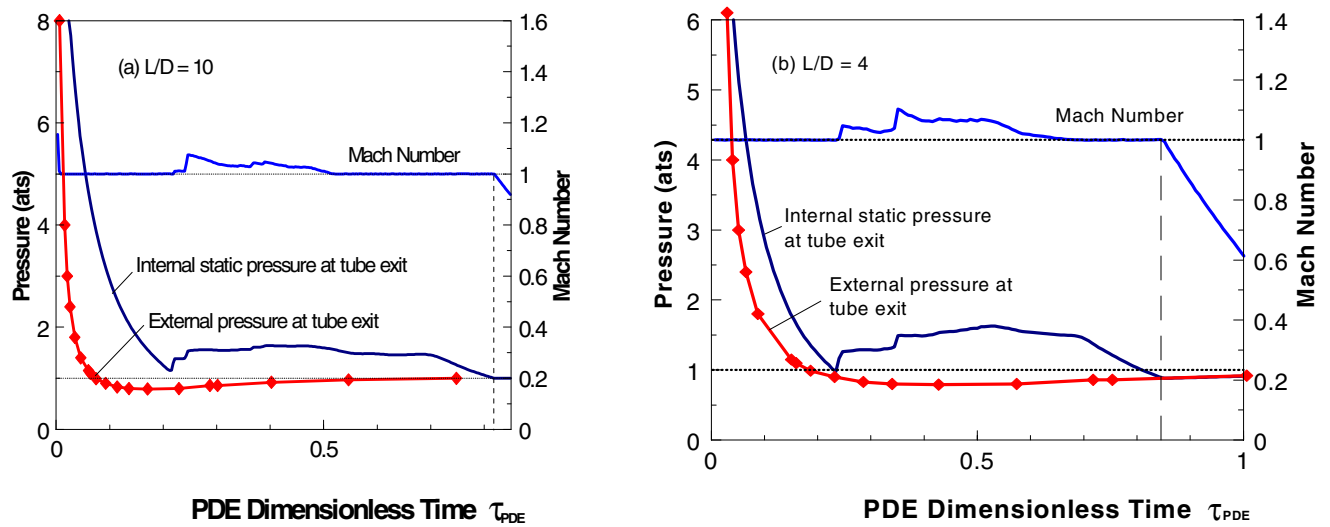
Figure 8. Diagram of the Half-Riemann problem.

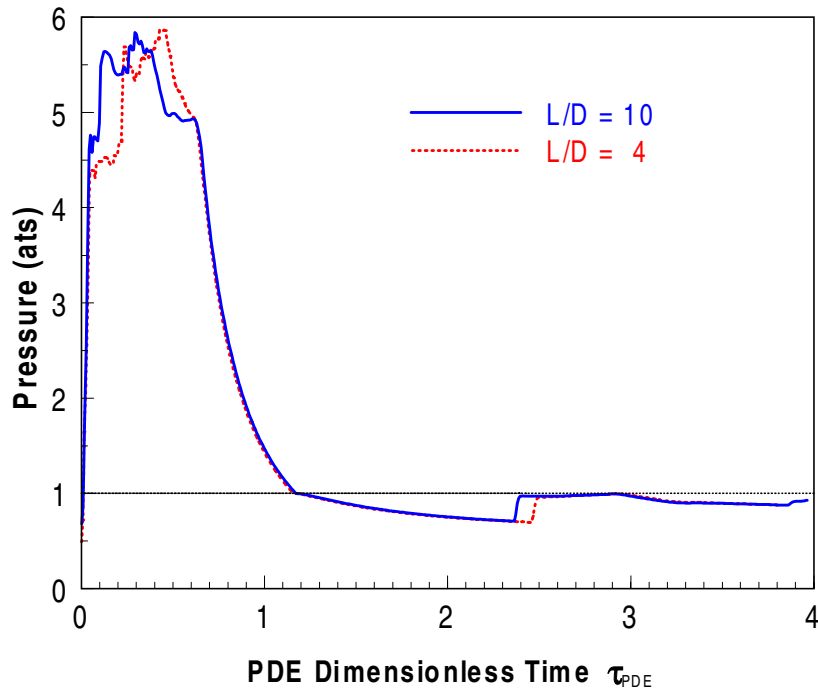Figure 9. Mach number and pressures at tube exit for (a) L/D = 10, and (b) L/D = 4.

Figure 10. Pressure on the front wall of the detonation tube versus τ_{PDE}, for L/D = 10, and L/D = 4.

Discussion

The present calculations indicate a reduction of thrust when the external pressure is taken into account, whereas Kailasanath (2001) found an increase in thrust. This is because the present results show the external pressure being atmospheric or less when the exit flow becomes subsonic, with a decrease in thrust when it is subatmospheric. Kailasanath used different decay times, with an exit pressure entirely above atmospheric, and found that the longer the pressure took to decay, the greater the thrust. Even his shortest decay time was longer than the decay time calculated here. As pointed out by Kailasanath, a long decay time might be achieved with a nozzle, and this seems to be a desirable technique.

The experiments of Ungut et al. (1984) were performed with detonations leaving a tube and traveling into the same detonable mixture as was in the tube initially. Thus, on the centerline, where the emerging wave is still planar to a distance of 0.7D, it is still a detonation. On the other hand, in a pulsed detonation engine, the wave will leave the tube, and propagate in air. The detonation will extinguish as soon as the detonation reaches the air-mixture interface. This is a different case from the experiments of Ungut et al., and may require a different prescription for the blast wave strength. At present, there is no basis for a different prescription. However, one would expect the blast wave to be weaker in this case, resulting in a more rapid pressure decay.

Conclusions

A blast wave model, based on the spherical shock being characterized by having half the Chapman-Jouguet velocity at a radius of 1.3 times the detonation tube diameter, appears to give realistic values for the externally imposed pressure distribution with time for an isolated detonation tube. However, it is only for tubes with length to diameter ratios less than 10 that this pressure distribution causes a result any different from a calculation using a constant pressure boundary condition. For lower values of length to diameter ratio, an effect on the thrust is noticed; the effect is to reduce the thrust slightly. The appropriate dimension for the pressure decay is the tube diameter.

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Appendix A: Comment on the shock energy

The prescription offered above that the emerging wave is characterized by having a velocity of half the detonation velocity, at a radius of 1.3 D must be related to the chemical energy liberated by the detonation, E_{chem}. Certainly the energy of the blast wave, E, can not be greater than E_{chem}, which is simply (Zitoun and Desbordes 1999):

$$E_{chem} = Q \, \rho_1 \, \pi \, D^2 \, \ell/4 \tag{A1}$$

In order to obtain a value for E from the prescription above, the strong shock form (Taylor 1950) of equation 1,

$$p_s \, / \, p_2 = 0.155 \, \lambda_s^{-3} \tag{A2}$$

is used together with the normal shock relations, in strong shock form,

$$p_s \, / \, p_2 \approx 2 \, \gamma_2 \, M_s^2 \, /(\gamma_2 + 1) \tag{A3}$$

Equating equations A2 and A3, and inserting the values at R_0 of $M_s = V_{cj}/ 2 \, a_2$, and $R_0 = 1.3$ D, and recalling that

$$\lambda_s = R_0 \, / \, \varepsilon = 1.3 \, D/(E/p_2)^{1/3} \tag{A4}$$

it is found that:

$$E = \rho_2 \, (1.3 \, D)^3 \, V_{cj}^2 \, /[\, 2 \, (\gamma_2 + 1) \, 0.155] \tag{A5}$$

The detonation velocity is related to the detonation enthalpy, Q, via (Zitoun and Desbordes 1999):

$$V_{cj}^2 = 2 \, (\gamma_3^2 - 1) \, Q \tag{A6}$$

So that:

$$\begin{aligned}
E &= \rho_2 \, Q \, D^3 \, [1.3^3 \, (\gamma_3^2 - 1) \, / \, 0.155(\gamma_2 + 1)] \\
&= 2.07 \, (\pi/4) \, \rho_2 \, Q \, D^2\ell \, (D/\ell) \\
&= 2.07 \, E_{chem} \, (D/\ell) \, (\rho_2 \, /\rho_1)
\end{aligned} \tag{A7}$$

As stated above, E cannot be greater than E_{chem}, implying that the simple prescription breaks down for ℓ/D less than 2.07 (assuming $\rho_2 \approx \rho_1$). The experiments of Ungut et al. (1984) were performed at $\ell/D = L/D = 13.4$ or greater, so that $E < E_{chem}$ for their results. Since the blast wave pressure, and radius scale with the cube root of E, and are therefore insensitive to the exact value of E, it is probably equally valid to use $E = E_{chem}$, as did Zitoun and Desbordes (1999). However, one would expect that there would be some losses which would make $E < E_{chem}$. This is why the experiments of Ungut et al. were used to determine E in this work.

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In one-dimensional calculations of pulsed detonation engine (PDE) performance, the exit boundary condition is frequently taken to be a constant static pressure. In reality, for an isolated detonation tube, after the detonation wave arrives at the exit plane, there will be a region of high pressure, which will gradually return to ambient pressure as an almost spherical shock wave expands away from the exit, and weakens. Initially, the flow is supersonic, unaffected by external pressure, but later becomes subsonic. Previous authors have accounted for this situation either by assuming the subsonic pressure decay to be a relaxation phenomenon, or by running a two-dimensional calculation first, including a domain external to the detonation tube, and using the resulting exit pressure temporal distribution as the boundary condition for one-dimensional calculations. These calculations show that the increased pressure does affect the PDE performance. In the present work, a simple model of the exit process is used to estimate the pressure decay time. The planar shock wave emerging from the tube is assumed to transform into a spherical shock wave. The initial strength of the spherical shock wave is determined from comparison with experimental results. Its subsequent propagation, and resulting pressure at the tube exit, is given by a numerical blast wave calculation. The model agrees reasonably well with other, limited, results. Finally, the model was used as the exit boundary condition for a one-dimensional calculation of PDE performance to obtain the thrust wall pressure for a hydrogen-air detonation in tubes of length to diameter ratio (L/D) of 4, and 10, as well as for the original, constant pressure boundary condition. The modified boundary condition had no performance impact for values of L/D > 10, and moderate impact for L/D = 4.

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